Design of Ferrite Differential Phase Shift Sections

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Ferrite differential phase shift sections are useful as nonreciprocal, circular polarizers in dual-mode phase shifters, as half-wave plates in precision rotary-field phase shifters, and in other devices where non-reciprocal birefringence is desirable. This paper presents a simple design model that provides accurate estimates of the differential phase shift and frequency dispersion of sections using transverse quadrupole magnetic field biasing in a circular waveguide completely filled with ferrite.

I. Introduction

Differential phase shift sections are an important ingredient in the dual-mode reciprocal, latching ferrite phase shifter, performing the function of a nonreciprocal circular polarizer [1]. The same basic geometry has also been used in the ferrite rotary-field phase shifter, where a magnetically rotatable half-wave plate is used to provide a very accurate relationship between phase shift and control current [2]. Design of these sections has in the past been carried out empirically because of the lack of a tractable computational model.

The basic geometry is that of a circular waveguide completely filled with ferrite as shown in cross-section in Figure 1.

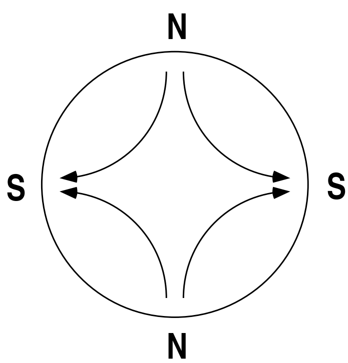

Fig. 1 Cross-sectional arrangement for differential phase shift section

Differential phase shift between orthogonally polarized quasi-TE_{11} modes is obtained by applying a transverse quadrupole field to the ferrite [3]. The principal axes of the differential phase shift are determined by the orientation of the poles of the biasing field. A frequency dispersion exists that depends on the guide diameter, ferrite material properties, and operating frequency, with the differential phase shift decreasing as frequency is increased. The computational model presented below allows accurate prediction of differential phase shift and frequency dispersion for this type of section with very simple arithmetic.

II. Computational Model

For a fully-filled guide with transverse magnetization, the nonreciprocal differential phase shift $\Delta\phi$ is given by a relationship of the form,

$$\Delta\phi = A\phi_o \frac{f_c}{f}\frac{\kappa}{\mu} \tag{1}$$

where κ and μ are effective values of the permeability tensor for the magnetized ferrite, f_c is the cutoff frequency of the waveguide, f is the operating frequency, ϕ_o is the insertion phase for a TEM wave propagating through the same length in the same medium, and A is a proportionality factor accounting for relative effectiveness of the waveguide and bias field distributions. For example, A can be computed as equal to $4/\pi$ for a simple rectangular waveguide with equal and opposite uniform bias field level over the right and left halves of the guide. The mathematics leading to equation (1) have been derived on the basis of a transmission line equivalent-circuit model [4].

The insertion phase ϕ_o can be expressed as

$$\phi_o = \beta_o l_p = \frac{360 f \sqrt{\mu_r \epsilon_r}\, l_p}{c} \quad \text{degrees} \tag{2}$$

with l_p defined as the polarizer length; furthermore,

$$f_c = \frac{c}{\lambda_c \sqrt{\mu_r \epsilon_r}} \tag{3}$$

and for a circular waveguide $\lambda_c = 1.705$ d, giving

$$\Delta\phi \quad 211.1\ A \frac{l_p}{d}\frac{\kappa}{\mu} \quad \text{degrees} \tag{4}$$

where d is the guide diameter.

When the rod is magnetized by a weak applied field, κ and μ can be approximated by [5]

$$\kappa \approx \frac{M}{M_s}\frac{\omega_m}{\omega} \tag{5}$$

$$\mu \approx \mu_i + (1-\mu_i)\frac{\tanh\left(1.25\ M / M_s\right)}{\tanh\left(1.25\right)} \tag{6}$$

where μ_i is the initial permeability, given by

$$\mu_i \approx \frac{1}{3} + \frac{2}{3}\sqrt{1-\left(\frac{\omega_m}{\omega}\right)^2} \tag{7}$$

If the applied field is large enough to saturate the rod completely, then κ and μ are more appropriately taken as

$$\kappa \approx -\frac{M}{M_s}\left(\frac{\omega\ \omega_m}{\omega_o^2 - \omega^2}\right) \tag{8}$$

$$\mu \approx 1\ \frac{\omega\ \omega_m}{\omega_o^2 - \omega^2} \tag{9}$$

In the above equations ω is the operating radian frequency, ω_m is the material characteristic radian frequency equal to the product of the gyromagnetic ratio and the saturation moment, and ω_o is the resonance frequency given by Kittel's equation,

$$\omega_o = \sqrt{\left\{\gamma H_o + \left(N_x - N_z\right)\omega_m\right\}\left\{\gamma H_o + \left(N_y - N_z\right)\omega_m\right\}} \tag{10}$$

Here γ is the gyromagnetic ratio, H_o is the externally applied field (z-directed), and N_x, N_y and N_z are demagnetizing factors for the ferrite shape.

The polarizer sections of concern here are usually biased near the "knee" of the magnetization curve to achieve a significant interaction with moderate mmf. This operating point does not properly conform to either the "weak-bias" or "strong bias" cases described above. However, experimental studies have yielded the following observations:

1. The coefficient A is numerically equal to about 1.23.
2. The "weak-bias" model fits experimental data reasonably well for $M/M_s \leq 0.25$. Above this level, a smooth transition begins toward the "strong-bias" model.
3. The "strong-bias" model gives a good fit to experimental data for M/M_s values near the "knee" of the magnetization curve.

Proceeding on this basis, take $H_o = \omega_m\ M/M_s$ in equation (10) and note that for a long rod transversely magnetized, with the x direction along the rod axis,

$$N_x = 0;\ N_y = N_z = 1/2 \tag{11}$$

Then substitute into equation (10) to get

$$r = \frac{\omega_o}{\omega_m} = \sqrt{\frac{M}{M_s}\left(\frac{M}{M_s} - \frac{1}{2}\right)} \tag{12}$$

Now it is possible to form the ratio κ/μ by substituting back into equations (8) and (9), and after a little bit of manipulation to get

$$\frac{\kappa}{\mu} \approx \frac{M}{M_s}\ \frac{m}{\left\{1 - m^2 r\ (1 + r)\right\}} \tag{13}$$

where $m \equiv \omega_m/\omega$ and r is defined by equation (12) above.

Then using A = 1.23, equation 4 for $\Delta\phi$ becomes,

$$\Delta\phi = 260.2\ \frac{l_p}{d}\ \frac{M}{M_s}\ \frac{m}{\{1 - m^2 r\ (1 + r)\}} \tag{14}$$

Normalized curves relating differential phase shift to the m and M/M_s ratios have been plotted in Figure 2 using this relationship.

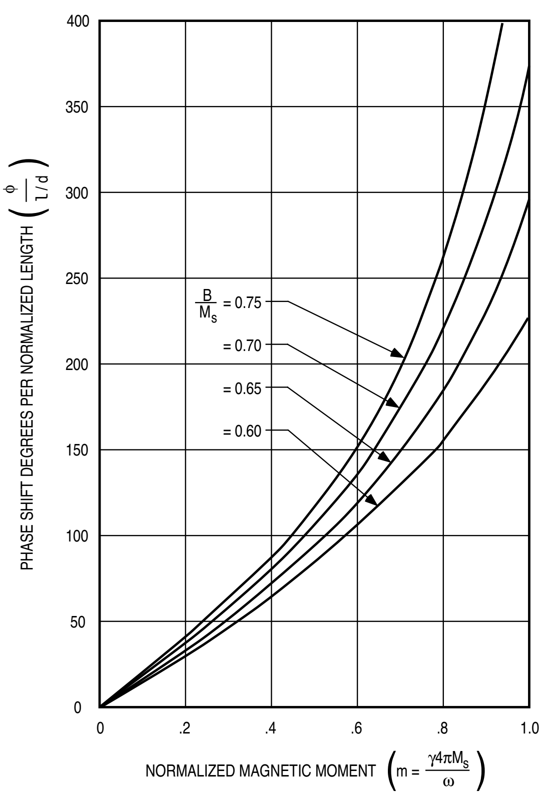

Fig. 2 Performance of differential phase shift section

III. Experimental Verification and Design Data

In most commonly used ferrites, the value M/Ms at the knee" of the magnetization curve is in the range 0.60 to 0.75. For example, in the case of lithium-titanium ferrite, a value of 0.75 is appropriate, so that

$$r = \sqrt{.75\ (.75 - .5)} = 0.433 \tag{15}$$

and

$$\Delta\phi = 195.1\ \frac{l_p}{d}\ \frac{m}{1 - 0.62m^2} \tag{16}$$

This expression gives results in good agreement with experimental data as plotted in Figure 3 for a C-Band circular polarizer. Similar agreement has been obtained at other frequency bands with other materials.

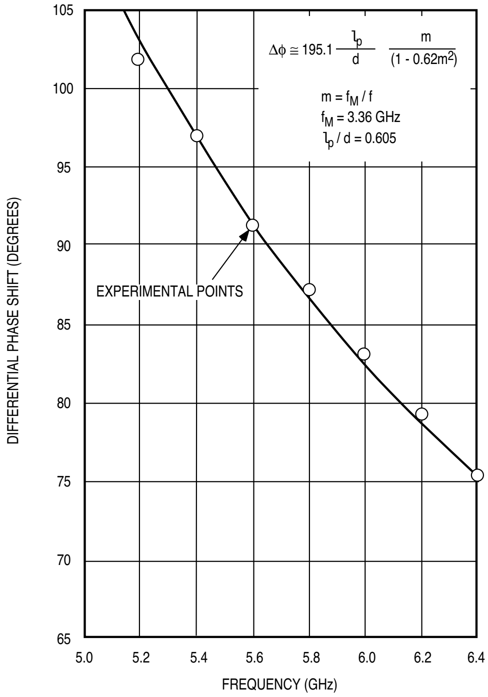

Fig. 3 Polarizer dispersion at C-band

Solving equation (14) for ℓ_p, with $\Delta\phi = 90°$,

$$\ell_p \approx 0.346 \frac{M_s}{M} d \frac{\{l - m^2 r(l + r)\}}{m} \tag{17}$$

Using this expression, a family of curves of normalized polarizer length has been plotted as a function of the m and M_r/M_s ratios in Figure 4. Next, Figure 5 shows curves of the fractional deviation from nominal differential phase, resulting from frequency dispersion, as a function of fractional bandwidth calculated in equation (14).

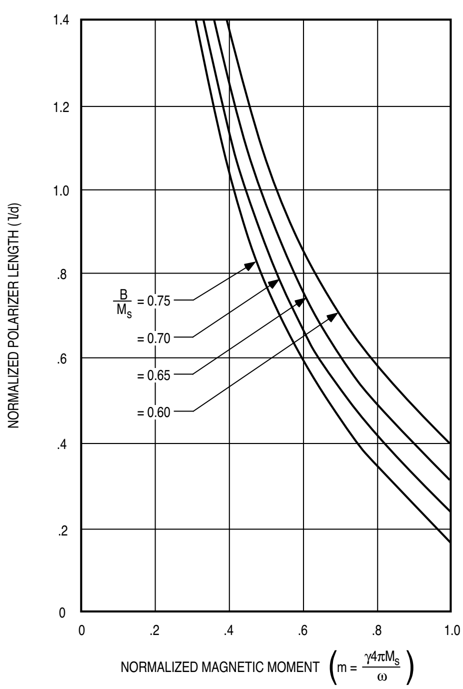

Fig. 4 Design data for ferrite circular polarizers

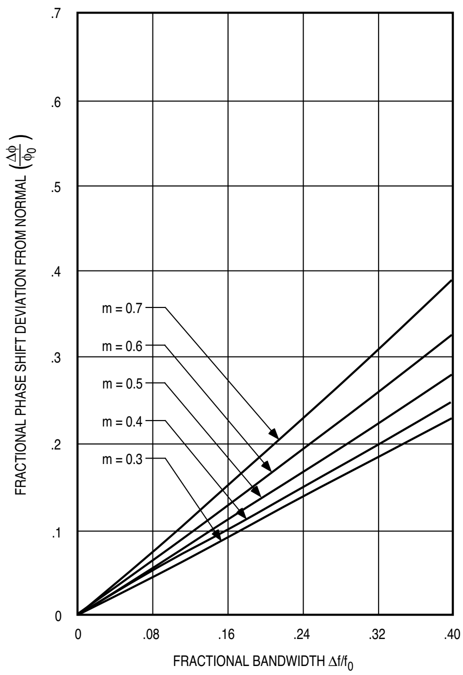

Fig. 5 Frequency-dispersive deviation of differential phase shift

IV. Conclusion

The simple algorithm presented here provides a useful and accurate method for calculating the differential phase shift available from a circular waveguide completely filled with ferrite. It also predicts that the frequency dispersion of the differential phase shift will depend on the saturation moment of the ferrite material, becoming greater with larger moment. In addition, the model predicts that the differential phase shift will be independent of the dielectric constant of the ferrite material. Neither of these other phenomena were postulated *a priori*, and yet, both have been observed experimentally on a wide variety of differential phase shift sections at different frequencies and using materials of significantly different dielectric constant.

V. References

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[5] J. J. Green, C. E. Patton and F. Sandy, "Microwave properties of partially magnetized ferrites," Rome Air Development Center, Rome, N.Y., Final Report RADC-TR-68-312, August 1968.